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3.1 - Simple Interest

Suppose you make a deposit or investment of P dollars or you take out a loan of P dollars that amount P is called the principal.

All of these things have an interest rate attached to them, essentially rent on the money, which is paid as interest.

Simple Interest

Simple interest is computed as

$$I = Prt$$

where

I = interest

P = principal

r = annual simple interest rate
(written as a decimal)

t = time in years

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①

Ex: Suppose you deposit \$2000 into a savings account with a simple interest rate (annual) of 6%. How much interest will accrue after 6 months?

Sol: 6 months = $\frac{1}{2}$ years so $t = \frac{1}{2} = 0.5$
interest is 6% so $r = 0.06$

The principal is $P = 2000$
The interest accrued is then

$$I = 2000(0.06)(0.5) = 60$$

So \$60 would be accrued after 6 months.

Often we might be more curious about how much will be in the account or how much will be owed on the loan after a certain period. This amount is called the future value.

Future Value

$$A = P + I = P + Prt$$
$$= P(1 + rt)$$

A = future value, P = principal or present value
 r = annual simple interest rate, t = time in years

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Ex Suppose you take out a \$10,000 loan at a simple annual interest rate of 3.2%. How much would be due on the loan after 10 months?

Sol: Principal = $P = 10000$
interest rate $r = 0.032$
 $t = 10 \text{ months} = \frac{10}{12} \text{ years} = \frac{5}{6} \text{ years}$

$$\text{Future value} = 10000(1 + (0.032)(\frac{5}{6}))$$
$$\approx 10000(1.027) = \$10,266.67$$

We can also use this formula to predict what interest rate we need or how much principal to take out/deposit.

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Ex: You're looking to invest \$5,000 and make \$1,000 in interest after 2 years. What annual rate on your investment will you need to accomplish this?

Sol: Principal = 5000, Interest = 1000
so future value is $A = 6000$. The time is $t = 2$ years. So, we plug in and get

$$6000 = 5000(1 + r(2))$$

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We need to solve for r :

$$\begin{aligned} 6000 &= 5000(1+2r) \\ &= 5000 + 10000r \end{aligned}$$

Subtract 5000 from both sides:

$$1000 = 10000r \Rightarrow r = \frac{1000}{10000} = \frac{1}{10} = 0.1$$

So we need an annual rate of 10% to make \$1000 on a \$5000 investment in 2 years.

One often uses a brokerage firm when making investments, many of which charge you a fee based on the ^{*}transaction^{*} amount (principal).
^{*}both buying AND selling^{*}

6) Ex: Suppose a brokerage firm uses the following commission schedule:

<u>Principal</u>	<u>Commission</u>
Under \$3,000	\$25 + 1.8% of principal
\$3,000 - \$10,000	\$37 + 1.4% of principal
Over \$10,000	\$107 + 0.7% of principal

An investor purchases 450 shares at \$21.40 per share, keeps the stock for 26 weeks, then sells the stock for \$24.60 per share. What was the annual interest rate earned on the investment?

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Sol: The investor purchases
 $\$21.40(450) = \$9,630$ (principal)
worth of shares. So, the commission on this is
 $\$37 + 0.014(\$9,630)$
 $= \$37 + \$134.82 = \$171.82$

Thus the total investment is the price of
the stock plus the commission:

$$\$9,630 + \$171.82 = \$9,801.82.$$

The investor then sells the stocks for
 $\$24.60(450) = \$11,070.$

The commission on the sale is then
 $\$107 + 0.007(\$11,070)$
 $= \$107 + \$77.49 = \$184.49$

Thus the return on the investment is the
sale price minus the commission:

$$\$11,070 - \$184.49 = \$10,885.51$$

So, now we can use the future value formula
to find the annual interest rate:

$$A = 10885.51, P = 9801.82 \text{ (use total investment!)} \\ t = \frac{26 \text{ weeks}}{52 \text{ weeks}} = 0.5 \text{ years}$$

$$10885.51 = 9801.82(1 + 0.5r) \\ = 9801.82 + 4900.91r$$

$$\Rightarrow 1083.69 = 4900.91r$$

$$\Rightarrow r = \frac{1083.69}{4900.91} \approx 0.22112 = 22.112\%$$

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A common method for calculating interest on a credit card is to use the average daily balance method. As the name suggests, the average daily balance is computed, then the interest is computed on that.

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Ex: A credit card has an annual interest rate of 19.99% and interest is calculated using the average daily balance method. If the starting balance of a 30-day billing cycle is \$523.18 and purchases of \$147.98 and \$36.27 were made on days 12 and 25, respectively, while a payment of \$200 was credited on day 17, what will be the balance on the card at the start of the next billing cycle?

Let's first figure out the daily balances:

Day 1-11:	\$523.18	(11 days)
Day 12-16:	\$671.16	(5 days)
Day 17-24:	\$471.16	(8 days)
Day 25-30:	\$507.43	(6 days)

The sum of the 30 daily balances is:

$$11(523.18) + 5(671.16) + 8(471.16) + 6(507.43) = \$15,926.89$$

The ADB is this divided by 30: \$530.90

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assuming a 360 day year

$$t = \frac{30}{360} = \frac{1}{12}$$

Now, we use the formula:

$$I = Prt = (1530.90)(0.1999)\left(\frac{1}{12}\right) = \$8.84$$

So, \$8.84 in interest is incurred.

To find the balance at the start of the next billing cycle, we add this interest to the remaining balance at the end of the last cycle:

So,

$$\text{New Balance} = \$507.43 + \$8.84 = \$516.27$$

↑
Day 30
balance

↑
Interest